Lösungen zu den Ferienübungen für die 9. Klasse (G8)

1. a) $\sqrt{5}+\frac{2}{\sqrt{5}}=\sqrt{5}+\frac{2\sqrt{5}}{5}=\left(1+\frac{2}{5}\right)\sqrt{5}=1{,}4\sqrt{5}$; b) $\frac{2-\sqrt{6}}{2+\sqrt{6}}=\frac{\left(2-\sqrt{6}\right)^2}{\left(2+\sqrt{6}\right)\left(2-\sqrt{6}\right)}=\frac{4-4\sqrt{6}+6}{4-6}=\frac{10-4\sqrt{6}}{-2}=2\sqrt{6}-5$

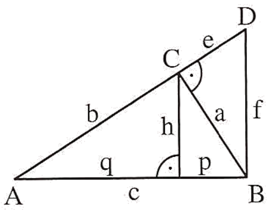
   c) $\frac{r-144}{\sqrt{r}-12}=\frac{\left(\sqrt{r}-12\right)\left(\sqrt{r}+12\right)}{\sqrt{r}-12}=\sqrt{r}+12$; d) $\frac{a-2\sqrt{a}+1}{\sqrt{a}-1}=\frac{\left(\sqrt{a}-1\right)^2}{\sqrt{a}-1}=\sqrt{a}-1$

2. a) $\sqrt{0{,}01p^2-0{,}6pq+9q^2}=\sqrt{(0{,}1p-3q)^2}=|0{,}1p-3q|$; c) $\left(5^{-\frac{1}{8}}\right)^4=5^{-\frac{1}{2}}=\frac{1}{\sqrt{5}}$

   b) $\sqrt{8r^4s^3}\cdot\sqrt{12r^3s^3}:\sqrt{4rs^2}=\sqrt{96r^7s^6:4rs^2}=\sqrt{24r^6s^4}=2\sqrt{6}\,r^3s^2$; d) $\sqrt[3]{\frac{1}{3}}:\sqrt[4]{3}=3^{-\frac{1}{3}}:3^{\frac{1}{4}}=3^{-\frac{7}{12}}$

3. p = $\frac{h^2}{q}=3\frac{1}{8}\,\text{cm}$ c = p+q = 11,125 cm b = $\sqrt{q^2+h^2}\approx 9{,}4\,\text{cm}$

   a = $\sqrt{p^2+h^2}\approx 5{,}9\,\text{cm}$ e = $\frac{a^2}{b}\approx 3{,}7\,\text{cm}$ f = $\sqrt{(b+e)e}\approx 7{,}0\,\text{cm}$



4. f (x) = $0{,}5x^2 - x + 0{,}75 = 0{,}5\,[x^2 - 2x + 1{,}5] =$
   $= 0{,}5\,[x^2 - 2x + 1 - 1 + 1{,}5] = 0{,}5\,[\,(x-1)^2 + 0{,}5] =$
   $= 0{,}5\,(x-1)^2 + 0{,}25$ ➔ S(1/0,25)

5. $f(x) = a(x+2)^2 - 2$; $f(0) = 1$ ➔ $4\cdot a - 2 = 1$ ➔ $a = \frac{3}{4}$ ➔ $f(x) = \frac{3}{4}(x+2)^2-2=\frac{3}{4}x^2+3x+1$

   $g(x) = a(x-1)^2 + 3$; $g(-1) = -2$ ➔ $4\cdot a+3 = -2$ ➔ $a = -\frac{5}{4}$ ➔ $g(x) = -\frac{5}{4}(x-1)^2+3=-\frac{5}{4}x^2+\frac{5}{2}x+\frac{7}{4}$

   Schnitt: f(x) = g(x) ➔ $\frac{3}{4}(x+2)^2-2=-\frac{5}{4}(x-1)^2+3$ ➔ $3(x+2)^2-8=-5(x-1)^2+12$

   $3x^2+12x+4=-5x^2+10x+7$ ➔ $8x^2+2x-3=0$ ➔ $x_{1/2}=\frac{-2\pm\sqrt{2^2-4\cdot8\cdot(-3)}}{2\cdot8}=\frac{-2\pm10}{16}$

   $S_1\left(-\frac{3}{4}/-\frac{53}{64}\right)$; $S_2\left(\frac{1}{2}/\frac{43}{16}\right)$

6. a) $\sqrt[4]{12x^2}=2$ ➔ $12x^2 = 2^4$ ➔ $x^2=\frac{16}{12}=\frac{4}{3}$ ➔ $|x| = \sqrt{\frac{4}{3}}=\frac{2}{3}\sqrt{3}$ ➔ $x_1=-\frac{2}{3}\sqrt{3}$; $x_2=\frac{2}{3}\sqrt{3}$

   b) $6z^2 - 2z = 0$ ➔ $2z\,(3z-1) = 0$ ➔ $z_1 = 0$, $z_2 = \frac{1}{3}$

   c) $3y^2 - 8y - 3 = 0$; $y_{1/2}=\frac{-(-8)\pm\sqrt{(-8)^2-4\cdot3\cdot(-3)}}{2\cdot3}=\frac{8\pm\sqrt{64+36}}{6}=\frac{8\pm10}{6}$; $\mathbf{y_1}=\frac{-2}{6}=-\frac{1}{3}$; $\mathbf{y_2}=\frac{18}{6}=3$

   d) $x^4 = 12 - x^2$; $x^4 + x^2 - 12 = 0$; <u>Substitution: $x^2 = y$</u>

   $y^2 + y - 12 = 0$; $y_{1/2}=\frac{-1\pm\sqrt{1^2-4\cdot1\cdot(-12)}}{2\cdot1}=\frac{-1\pm\sqrt{49}}{2}=\frac{-1\pm7}{2}$

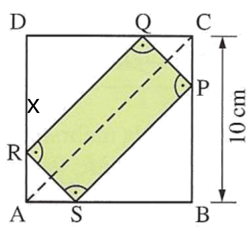
   $y_1 = (-4)$ ➔ <u>Resubstitution</u>: $x^2 = (-4)$ ***nicht def.***

   $y_2 = 3$ ➔ <u>Resubstitution</u>: $x^2 = 3$; $\mathbf{x_1=-\sqrt{3}}$, $\mathbf{x_2=\sqrt{3}}$

7. Es gilt: $x=\overline{DR}=\overline{DQ}=\overline{BP}=\overline{BS}$

   $A = A_{Qu} - 2\,A_{\text{Dreieck groß}} - 2\,A_{\text{Dreieck klein}}$

   $A(x) = 100 - 2\cdot0{,}5\cdot x^2 - 2\cdot0{,}5\cdot(10-x)^2 = 100 - x^2 - (100 - 20x + x^2) =$
   $= -2x^2 + 20x = -2\,[x^2 - 10x + 25 - 25] = -2\,(x-5)^2 + 50$



   S (5 | 50) Den maximalen Flächeninhalt y = 50 cm² erhält man für x = 5 cm; das Rechteck mit maximalem Flächeninhalt ist also ein Quadrat !

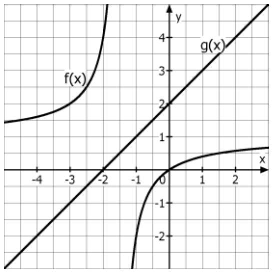
8. a) A (1 | 0) $\in G_f$ ➔ I) $0 = a\cdot 1^2 + b\cdot 1 + c = a + b + c$
B (2 | 1) $\in G_f$ ➔ II) $1 = a\cdot 2^2 + b\cdot 2 + c = 4a + 2b + c$
C (4 | –3) $\in G_f$ ➔ III) $-3 = a\cdot 4^2 + b\cdot 4 + c = 16a + 4b + c$

Aus I) : $c = -a - b$; in II): $1 = 3a + b$ (II')
in III): $-3 = 15a + 3b$ | :3
$-1 = 5a + b$ (III')

III)' – II)' $-2 = 2a$ **a = (–1)** *(Additionsverfahren)*
a in II)' $1 = 3\cdot(-1) + b$ **b = 4**
a, b in I)' $c = -(-1) - 4$; **c = (–3)** **$f(x) = -x^2 + 4x - 3$**

b) Nullstellen: $x_1 = -2$; $x_2 = 1$ => $f(x) = a(x - x_1)(x - x_2) = a(x + 2)(x - 1)$

C (0 | 3) $\in G_f$ => $3 = a(0 + 2)(0 - 1)$; $3 = -2a$; $a = -1{,}5$
➔ **$f(x) = -1{,}5(x + 2)(x - 1) = -1{,}5\cdot x^2 - 1{,}5\cdot x + 3$**

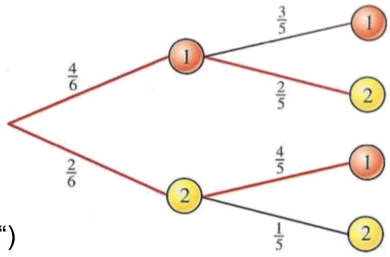


9. $\frac{2x}{2x+3} = x + 2$ ➔ $2x = (2x + 3)(x + 2)$ ➔ $2x = 2x^2 + 7x + 6$

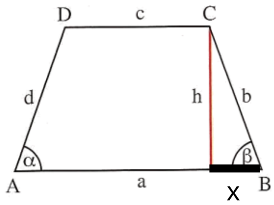
➔ $0 = 2x^2 + 5x + 6$ ➔ $x_{1/2} = \frac{-5 \pm \sqrt{5^2 - 4\cdot 2\cdot 6}}{2\cdot 2} = \frac{-5 \pm \sqrt{-23}}{4}$ ***nicht def.***

G_f und G_g besitzen **keinen** Schnittpunkt !

10. $\Omega = \{11; 12; 21; 22\}$; $A = \{12; 21\}$
$P(A) = \frac{4}{6}\cdot\frac{2}{5} + \frac{2}{6}\cdot\frac{4}{5} = \frac{16}{30} = \frac{8}{15}$



11. **Gegen**ereignis: Frau Schnell steht an **keiner** Ampel.
P(„mind. eine Ampel") = 1 – P(„keine Ampel")
$= 1 - \left(\frac{1}{2}\right)^3 = \frac{7}{8} = 87{,}5\ \%$



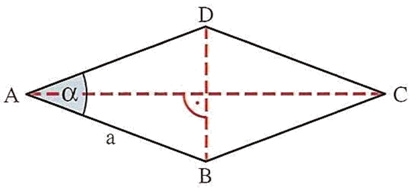
12. $\sin\beta = \frac{h}{b} \Rightarrow \beta = \sin^{-1}\left(\frac{h}{b}\right) = \sin^{-1}\left(\frac{2}{3}\right) \approx 41{,}8°$

$\tan\beta = \frac{h}{x} \Rightarrow x = \frac{h}{\tan\beta} \approx 5{,}6$ cm

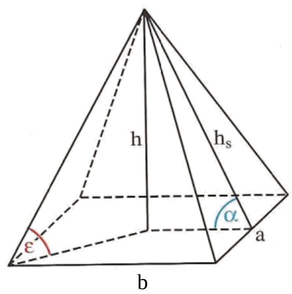
$a = c + 2x \approx 15{,}7$ cm ; $d = b = \sqrt{h^2 + x^2} \approx 7{,}5$ cm ; $A = \frac{1}{2}(a + c)\cdot h \approx 50{,}5\ cm^2$

13. $\sin\left(\frac{\alpha}{2}\right) = \frac{0{,}5\,\overline{BD}}{a} = \frac{\overline{BD}}{2a} \Rightarrow \overline{BD} = 2a\cdot\sin\left(\frac{\alpha}{2}\right) \approx 5{,}2$ cm

$\cos\left(\frac{\alpha}{2}\right) = \frac{0{,}5\,\overline{AC}}{a} = \frac{\overline{AC}}{2a} \Rightarrow \overline{AC} = 2a\cdot\cos\left(\frac{\alpha}{2}\right) \approx 6{,}9$ cm



14. a) h = 10 cm; Grundfläche: a = 8 cm; b = 15 cm
$V = \frac{1}{3}\,G\,h = \frac{1}{3}\,ab\,h = 400\ cm^3$
$O = G + 2A_{\text{Dreieck groß}} + 2A_{\text{Dreieck klein}} = a\cdot b + 2\cdot 0{,}5\cdot b\cdot h_b + 2\cdot 0{,}5\cdot a\cdot h_a$
$= ab + b\cdot\sqrt{h^2 + \left(\frac{1}{2}a\right)^2} + a\cdot\sqrt{h^2 + \left(\frac{1}{2}b\right)^2} \approx 382\ cm^2$



b) $\tan\varepsilon = \frac{h}{0{,}5\sqrt{a^2 + b^2}}$ => $\varepsilon = \tan^{-1}\left(\frac{2h}{\sqrt{a^2 + b^2}}\right) \approx 49{,}6°$;

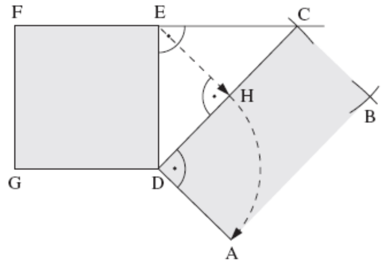
c) $\tan\beta = \frac{h}{0{,}5b} = \frac{2h}{b}$ ➔ $\beta = \tan^{-1}\left(\frac{2h}{b}\right) \approx 53{,}1°$; $\tan\alpha = \frac{h}{0{,}5a} = \frac{2h}{a}$ => $\alpha = \tan^{-1}\left(\frac{2h}{a}\right) \approx 68{,}2°$

Lösungen zu den Zusatzaufgaben:

15. a) $(3y-2x)(-2x-3y) = (-2x+3y)(-2x-3y) = 4x^2-9y^2$; b) $\left(\sqrt{3}\,s+t^2\right)^2 = 3s^2+2\sqrt{3}st^2+t^4$

c) $\left(\frac{1}{8}p^3-1\right)^2 = \frac{1}{64}p^6-\frac{1}{4}p^3+1$; d) $\sqrt{2x}\left(\sqrt{8xy}+\sqrt{6x^3}\right) = \sqrt{16x^2y}+\sqrt{12x^4} = 4x\sqrt{y}+2\sqrt{3}x^2$

16 C liegt auf [FE und auf k (D; 6 cm);
H ist der Fußpunkt des Lotes von E auf [DC];
A liegt auf einem Lot durch D zu [DC] und auf k (D; $\overline{DH}$)
B liegt auf k (A; $\overline{DC}$) und auf k (C; $\overline{DA}$)



17.

Ist Element aus:	N	Z	Q	R
$-\left(\sqrt{2}\right)^2 = -2$	/	ja	j ja	ja
$15,\overline{23} = 15\frac{23}{99}$	/	/	ja	ja
π	/	/	/	ja
$\sqrt{1{,}69}$ = 1,3	/	/	ja	ja
$6^{2,5} = 6^{\frac{5}{2}} = \sqrt{6^5} = 36\sqrt{6}$	/	/	/	ja

18. a) $\left(7^{-\frac{1}{6}}\right)^3 = 7^{-\frac{1}{6}\cdot 3} = 7^{-\frac{1}{2}} = \frac{1}{\sqrt{7}} = \frac{1}{7}\sqrt{7}$; b) $\sqrt[3]{\frac{1}{3}} : \sqrt[4]{3} = 3^{-\frac{1}{3}} : 3^{\frac{1}{4}} = 3^{-\frac{7}{12}}$

c) $\left(3x^3\right)^{-\frac{1}{4}}\cdot(27x)^{-\frac{1}{4}} = \left(81x^4\right)^{-\frac{1}{4}} = \frac{1}{\sqrt[4]{81x^4}} = \frac{1}{3x}$; d) $\sqrt{(-a-b)^2} = |-a-b| = |a+b|$

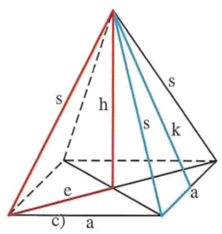
19. a) $y^2 = 0{,}25$ ➔ $|y| = \sqrt{0{,}25}$ ➔ $y_1 = -0{,}5$; $y_2 = 0{,}5$

b) $z^5 + 1024 = 0$ ➔ $z^5 = -1024$ ➔ $z = -\sqrt[5]{1024} = -4$ (Vorsicht: $\sqrt[5]{-1024}$ ist nicht definiert!)

c) $\sqrt[3]{4x} = 5$ ➔ $4x = 125$ ➔ $x = \frac{125}{4} = 31{,}25$

20. a = $\sqrt{G} = \sqrt{1225\text{ m}^2}$ = 35 m   Seitenfläche A = 2000 m² : 4 = 500 m²

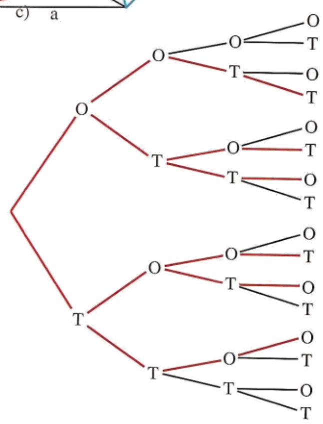
A = $\frac{1}{2}ak \;\Rightarrow\; k = \frac{2A}{a} = 28\frac{4}{7}\text{ m}$ h = $\sqrt{k^2-(0{,}5a)^2} \approx$ **22,6 m**



21. a) P ({OTTO}) = $\frac{2\cdot2\cdot1\cdot1}{4\cdot3\cdot2\cdot1} = \frac{4}{24} = \frac{1}{6}$

b) P ({OTTO}) = $\frac{2\cdot2\cdot2\cdot2}{4\cdot4\cdot4\cdot4} = \left(\frac{1}{2}\right)^4 = \frac{1}{16}$

c) Jeder Ast ist mit der Wsk. $\frac{1}{2}$ zu beschriften: P ({OTTO}) = $6\cdot\left(\frac{1}{2}\right)^4 = \frac{6}{16} = \frac{3}{8}$



22. d = 8,1 cm ➔ r = 4,05 cm ; V = 0,5 l = 0,5 dm³ = 500 cm³ ;

$V = r^2\cdot\pi\cdot h$ ➔ $h = \frac{V}{r^2\pi}$; $O = 2\cdot\pi\cdot r^2 + 2\pi r\cdot h = 2\pi r^2 + 2\pi r\cdot\frac{V}{r^2\pi} = 2\pi r^2 + \frac{2V}{r}$

Bedarf: $B = 115\,\%\cdot O = 1{,}15\cdot O = 1{,}15\left(2\pi r^2+\frac{2V}{r}\right) \approx 402{,}5\text{ cm}^2$

23. Bezeichnet man das gesuchte Volumen mit V_1 und die gesuchte Höhe mit h_1, so gilt: $h_1 = 0{,}5\,h$.

Für den dazugehörigen Radius r_1 gilt nach dem Strahlensatz: $\frac{r_1}{r} = \frac{h_1}{h} \Rightarrow r_1 = \frac{h_1}{h}\cdot r = \frac{0{,}5\cdot h}{h}r = 0{,}5r$

$\frac{V_1}{V} = \frac{\frac{1}{3}r_1^2\pi h_1}{\frac{1}{3}r^2\pi h} = \frac{(0{,}5r)^2\,0{,}5h}{r^2h} = \frac{0{,}125r^2h}{r^2h} = \frac{1}{8} = 12{,}5\%$ Bei halber Füllhöhe ist das Glas also zu 12,5 % gefüllt.